**Question 1 – Representation of Fuzzy Sets**

A fuzzy set in (classical set of objects, called the universe, whose generic elements are denoted ) is a set of ordered pairs:

* The of fuzzy set is the ordinary set
* The of fuzzy set is the ordinary set
* The set of all distinct numbers in [0, 1] that are employed as membership grades of the elements of in is called the *level set* of *A,* denoted by.
* The *support of* , denoted , is defined as the set of elements of *X* that have nonzero membership in *A*.
* The *core of* , denoted , is defined as the set of elements of *X* that have membership in *A* equal to 1.
* The membership function of *A* can be expressed in terms of the characteristic functions of its *s* according to the formula:

where (\*)

In the case of a discrete fuzzy set we have Notice that sup in (\*) means superior (for discrete fuzzy sets superior becomes the maximum) value of those obtained when multiplying the distinct values of in the level set with 1 or 0 (the value of – depending on whether an value belongs or not to the alpha-level set .

*Assume minimum and maximum operators for the intersection and union of fuzzy sets.*

1. Given any two fuzzy sets and , prove that the following properties hold:

Answer:

**1. proof of**

To prove that We will use the following two properties

* Property 1
* Property 2

Step 1 Solution: to prove

suppose . It is known that

Therefore,

min both A (x) and B (x) therefore,

we can say that

**Eq(1)**

Step 2 Solution to prove :

suppose therefore,

min

Min value is not known; however, both sets, A (x) and B(x) are

so,

and since

**Eq(2)**

According to **Eq(1) and Eq(2)** we can say the following;

**2. proof of we will use the following properties**

* Property 1
* Property 2

Step 1 Solution: prove

max

so,

Since it is known that

**Eq(3)**

Step 2 Solution: prove

suppose therefore,

It is known that

Hence,

It is also known that

It is not known which membership function have the max value. However,

both hence,

since . Therefore,

**Eq(4)**

According to the Eq(3) and Eq(4),

(8 marks)

1. How do and relate to the and the of *?*

(4 marks)

1. For the discrete fuzzy sets , obtain *,* andprovide all the distinct of *.*

(6 marks)

1. Show that formula (\*) is true for the discrete fuzzy set of c).

(5 marks)

**Question 2 – Fuzzy sets intersection, union and complement**

Consider the two discrete fuzzy sets:



In some sense the fuzzy sets and are complementary. However, the operator that has been used to make is obviously not the negation operator of Zadeh, *N*(*x*) = 1 - *x*.

1. Prove whether the operator that has been used to make from obeys the law of "double negation", i.e: whether .

(10 marks)

1. Determine the fuzzy sets and using the original definitions for the intersection ( and union (operations proposed by L.A. Zadeh in 1965.

(10 marks)

1. What should be the answers of and in case and were classical sets. Explain the difference in both results.

(5 marks)

**Question 3 – Decision making in a fuzzy environment**

Fuzziness can be introduced at several points in the existing models of decision making. Bellman and Zadeh in 1970 suggested a fuzzy model of decisions that must accommodate certain constraints and goals .

Suppose we must choose one of four different jobs *a, b, c,* and *d*, the salaries of which are given by the function such that:

* Our goal is to choose the job that will give us a high salary given the constraints that the job is interesting and within close driving distance.
* The first constraint of interest value is represented by the fuzzy set
* The second constraint concerning the driving distance to each job is defined by the fuzzy set
* The fuzzy goal *G* of a high salary is defined by the membership function

1. Provide a description of Bellman and Zadeh fuzzy model of decision making. (15 marks)
2. Which is the best job when applying Bellman and Zadeh’s fuzzy decision model? (10 marks)

**Question 4 – Type 2 defuzzification**

Write a short essay (about 2 pages not including bibliography & references) discussing the issue of computational complexity in relation to the defuzzification of type-2 fuzzy sets. Your answer should include definitions of *computational complexity* and *type-2 defuzzification* and an explanation of the concept of the *type-2 embedded set* (illustrated diagrammatically). It should also explain the strategy of Exhaustive Defuzzification, and why embedded sets give rise to the issue of computational complexity for Exhaustive Defuzzification. Choose a type-2 defuzzification method developed at De Montfort University (other than the Exhaustive Method), and briefly present the technique, using algorithmic notation, mathematical notation, and/or a diagram as appropriate. Lastly, if you were developing a type-2 FIS, would you use the method you have chosen? Explain your reasons.

(Worth 25 marks)

**This assessment covers Learning Outcomes 1 and 3 (LO1, LO3) on the module template:**

LO1: critically evaluate fuzzy logic approaches to solve computational problems exhibiting uncertainty and imprecision;

LO2: have a comprehensive understanding of the successful application of fuzzy logic to several problem domains and be capable of judging whether the fuzzy paradigm might be fruitful in a novel situation.